

A TECHNIQUE FOR THE RAPID CALCULATION OF DISTORTION EFFECTS  
IN VARACTOR PARAMETRIC AMPLIFIERS

by

Donald R. Chambers

David K. Adams

Stanford Research Institute

Menlo Park, California 94025

Varactor parametric amplifiers (paramps) are useful for obtaining low-noise gain over relatively large bandwidths at microwave frequencies. Although usually intended for use where small signals are to be amplified, paramps often work in environments where large signals also exist. Therefore, multi-signal performance must be considered. As with other types of amplifiers, paramps exhibit distortion in the form of gain saturation, cross modulation, and intermodulation. Often, it is desirable to be able to predict, without lengthy calculations, the distortion performance of a paramp in advance, before committing a design to hardware. In addition, it is useful to be able to relate distortion performance directly to the specifications of the nonlinear element (varactor) so that distortion effects can be considered at the earliest possible stage in the design.

Distortion effects in paramps can be expressed in terms of a few "external" parameters: (1) varactor specifications including cut-off frequency  $f_{co}$ , junction capacitance, and the exponent describing capacitance variation with voltage; (2) paramp small signal gain; (3) available powers of signals and pump; (4) signal and pump frequencies. Thus, it is possible to relate distortion directly to the nonlinear element through its specifications (Item (1) above) and through the operating conditions (Items (2), (3), and (4) above).

Distortion effects in a paramp are conveniently explained in terms of a series expansion of the nonlinear characteristic as follows:

$$C(V) = C(0) \left( 1 - \frac{V_b}{\varphi} \right)^{n-1} = C_0 + C_1(V - V_b) + C_2(V - V_b)^2 + \dots \quad (1)$$

where  $C(0)$  is the junction capacitance at zero bias,  $\varphi$  is the built-in or diffusion potential,  $V_b$  is the bias voltage,  $n$  is the capacitance exponent, ranging typically between 1/3 and 1/2, and  $C_k = (1/k!) \times [\partial^k C(V)/\partial V^k]_{V=V_b}$ . The linear term  $[C_1(V - V_b)]$  is the basis of the mixing action necessary for parametric amplification, but gives rise to some distortion. The quadratic  $[C_2(V - V_b)^2]$  and higher order terms are always sources of distortion.  $C_0$  denotes the average diode capacity in the absence of signals. When signals are applied, the even order terms in Eq. (1) cause a higher apparent diode capacity to arise, which is denoted by  $C'_0$ .

#### GAIN SATURATION

In a modestly high gain paramp, gain saturation occurs because of a pump circuit reaction to large input signals. The impedance presented to the pump by the varactor is constant for small signals, but a pump mismatch develops when large signals are applied and the effective pump voltage then becomes reduced. The result is mainly a drop in magnitude of the negative conductance  $-G$ , and secondly a detuning change in  $C'_0$ .

The threshold for gain saturation due to a reduction in  $(-G)$  can be given in the following simple expression

$$SSP = GF(f_s/f_p)APP \quad (2)$$

where SSP (Saturated Signal Power) is the available signal power required to produce 1 dB gain reduction; APP (Available Pump Power) is the pump power required to produce the given small signal gain;  $f_s/f_p$  is the ratio of (instantaneous) signal frequency to the pump frequency; and GF is a parameter plotted in Fig. 1. The gain factor, GF, depends mainly on the small signal gain and to a lesser extent on a dimensionless parameter,  $Q$ , also given in Fig. 1. It is not necessary to know  $Q$  accurately to obtain a rough estimate of the gain saturation because a maximum spread of 6 dB is indicated for all  $Q$  values.

Figure 1 can be used to predict the threshold of gain saturation in a given situation with minimum knowledge of the paramp design. In addition, the following general conclusions can be made.

1. Gain saturation is a property of the pump circuit and occurs before clipping or other mechanisms (that tend to produce signal harmonics) occur.

2. The saturation level varies inversely with paramp gain. Fig. 1 shows that for every 2 dB reduction in the small signal gain, a 3 dB increase in SSP is realized. Therefore, the paramp gain should not exceed that required to overcome second stage noise.

3. The large signal range of a paramp can be increased by designing with  $Q$  (Fig. 1) as close to 1 as possible.  $Q$  is a function of pump power.

4. Any signal in the paramp input band is effective in causing gain saturation, therefore, the paramp will saturate when the sum of all the signal powers falling in the paramp input band reaches the level denoted by SSP.

5. Detuning effects accompany gain saturation due to a reduction in  $C'_0$ . For  $Q > 1$ , detuning tends to raise (somewhat) the threshold predicted by Eq. (2), with the opposite effect for  $Q < 1$ . Also the paramp tuning shifts to higher frequency.

#### INTERMODULATION

The dominant in-band intermodulation in a paramp is third-order intermodulation in which either two or three in-band signals combine to produce spurious signals (for example, signals at three in-band frequencies can produce as many as nine in-band spurious frequencies by

third order intermodulation). The dominant in-band intermodulation products involve the pump circuit also, as did gain saturation. When signals at two or more frequencies appear in the input passband, sidebands appear in the pump circuit spaced from the pump by the input frequency differences. The pump sidebands are significant because of the gain associated with the up-conversion process in a reactive mixer. Contributions to pump sidebands occur in two ways, as follows: (1) A signal at  $f_a$  mixes with the idler (at  $f_{bb}$ ) of a second signal at  $f_b$  to produce a lower sideband ( $f_b > f_a$ ). The upper sideband is produced when the signal at  $f_b$  mixes with the idler (at  $f_{aa}$ ) of the signal at  $f_a$ . The sidebands are separated from the pump frequency by  $f_b - f_a$ . This contribution is equal for each pump sideband and comes about through a second-order process involving the coefficient  $C_1$ . (2) A third-order mixing process involving the coefficient,  $C_2$ , also contributes to the same pump sidebands. Mixing the signals at  $f_a$  and  $f_b$  (or idlers at  $f_{aa}$  and  $f_{bb}$ ) with the pump produces upper and lower sidebands spaced  $f_b - f_a$  from the pump frequency. In general, the third-order contribution to the pump sidebands will be greater than the second order contribution.

Once pump sidebands are produced, either by  $C_1$  or  $C_2$  or both, in-band intermodulation products arise from mixing between the pump sidebands and the idler of any in-band signal. The latter process involves only  $C_1$ , and it occurs in two specific situations of interest: Case A: The intermodulation product results from three different in-band signals. The pump sidebands from two of the signals mix with the idler of the third to produce an intermodulation product. This process is also important in producing cross-modulation effects. Case B: The intermodulation results from two different in-band signals. The pump sidebands from the two signals mix with the idler from one to produce an intermodulation product.

Estimates of intermodulation levels can be obtained in terms of the signal levels in the input band and the following design parameters: (1) the small-signal amplifier gain; (2) the diode capacitance coefficient ratios:  $C_1/C_0$ ,  $C_2/C_0$ , etc.; (3) the ratios of pump frequency to signal frequency; and (4) the available pump power (APP). It is convenient to express each intermodulation product in terms of an equivalent available source power at that frequency. Therefore, let AIP (Available Intermodulation Power) denote the available signal power that would be required from an external source to produce the intermodulation level observed in the input band if no other signals were present, and let  $ASP_x$  (Available Signal Power) denote the available signal power at frequency  $x$ . The intermodulation power delivered to the load is AIP times the amplifier gain. The available intermodulation power can be computed from the following formulas:

$$\underline{\text{Case A:}} \quad f_{\delta} = f_a + f_b - f_c \quad (3a)$$

$$AIP_{\delta} = 4K_3 ASP_a ASP_b ASP_c$$

$$\underline{\text{Case B:}} \quad f_{\delta} = 2f_a - f_b \quad (3b)$$

$$AIP_{\delta} = K_3 (ASP_a)^2 ASP_b$$

where  $K_3$  is the third order intermodulation coefficient and is usually expressed in dBm. Information for calculating  $K_3$  is given in Fig. 2,

and two steps are involved. One is to graphically determine  $F$  which depends on the paramp gain, and the other is to compute  $L$  using the diode and pump parameters given in Fig. 2.  $L$  depends upon the capacitance expansion coefficients involved, and expressions are given in Fig. 2 for  $L$  due to  $C_1$  alone and  $L$  due to  $C_2$ . Normally, the intermodulation due to  $C_2$  dominates, but when both the  $C_1$  and  $C_2$  contributions are significant it has been determined that the third order intermodulation powers can be computed separately and added.

For very large signals, the third-order intermodulation coefficient  $K_3$  saturates such that the intermodulation ratio ( $AIR_3/ASP_a$ , where  $ASP_a$  denotes the smaller of the contributing signal powers) will not significantly exceed unity for in-band signals. (Support for this conclusion can be obtained from the Manley-Rowe equations.) For  $Q > 1$ , gain saturation from the stronger intermodulating signal limits the intermodulation ratio to values less than unity. For  $Q < 1$ , the intermodulation ratio will saturate near unity.

JASIK LABORATORIES, INC.  
100 Shames Drive, Westbury, New York

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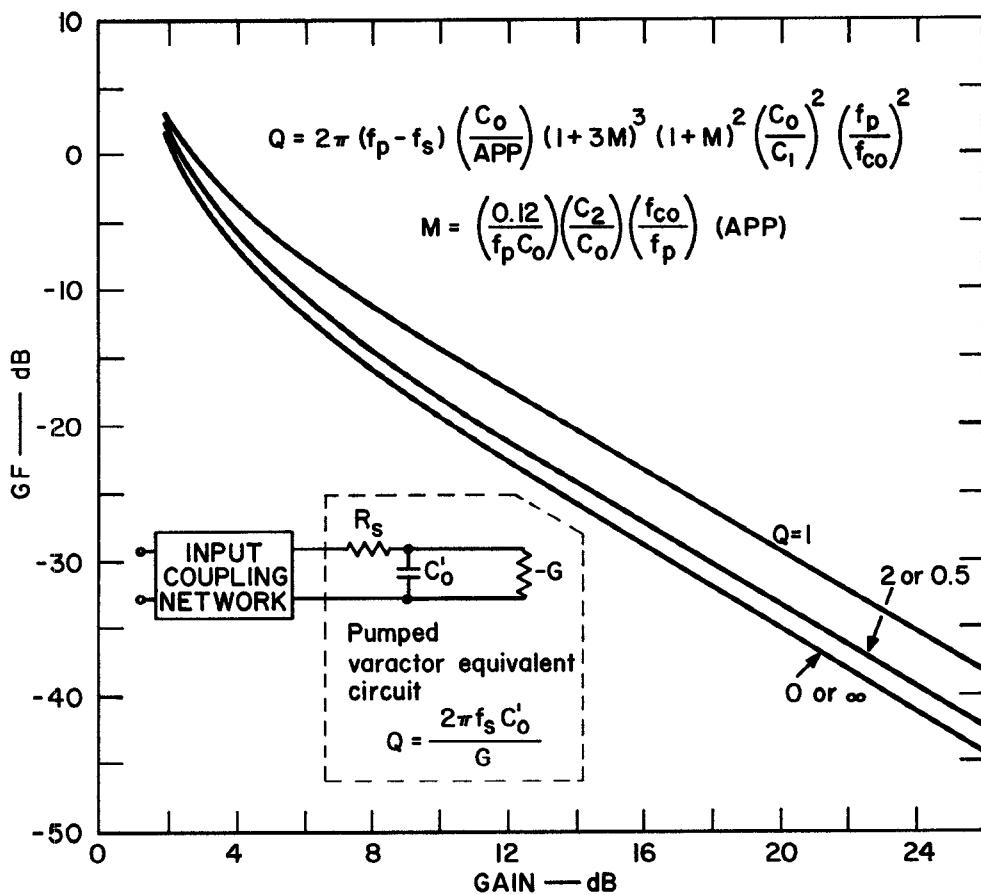


FIG. 1  
GENERAL PARAMETRIC AMPLIFIER  
GAIN SATURATION CHARACTERISTIC

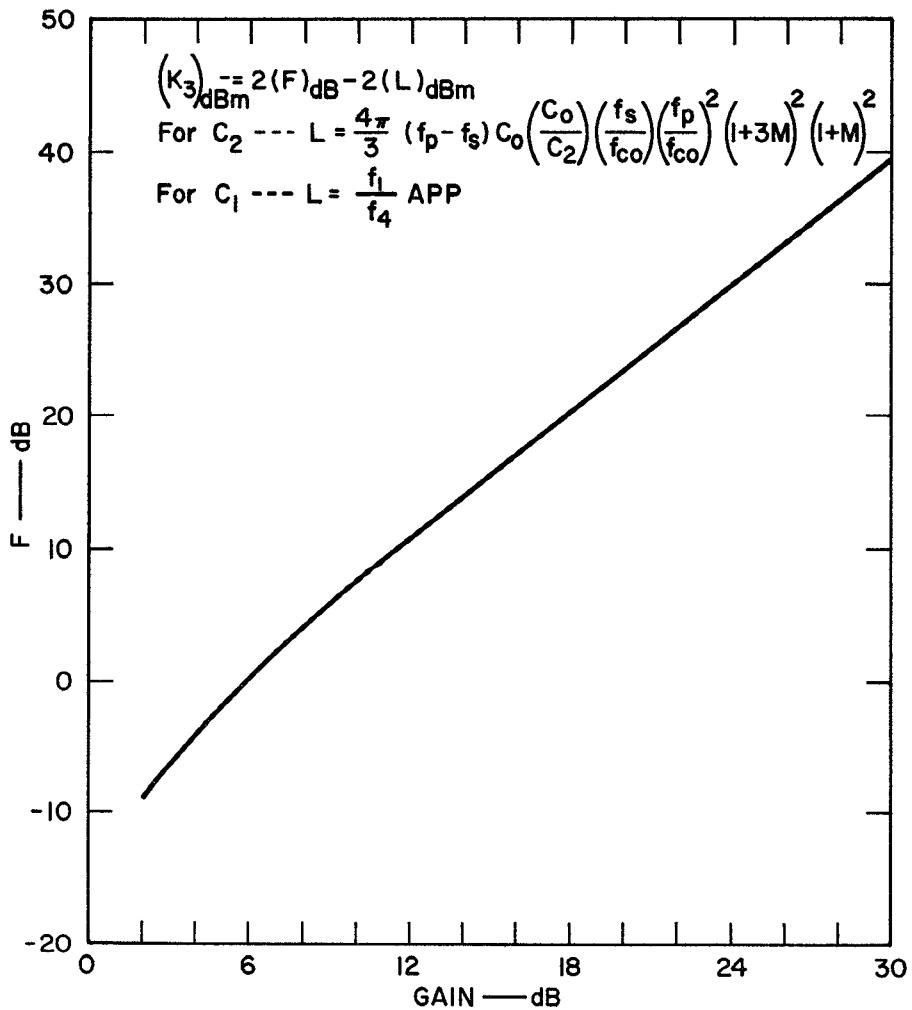


FIG. 2  
GENERAL PARAMETRIC AMPLIFIER  
INTERMODULATION CHARACTERISTIC